NONSTEADY WAVE PROCESSES IN AEROSOLS

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The propagation of steady-state shock waves and the nonsteady formation of a relaxation zone in the interaction of a wave with an aerosol cloud was analyzed in [1]. The same topic was addressed in the bibliography of this study within the framework of a continuum (C) model of heterogeneous media [2]. These investigations used general simplifying assumptions: motion is unidimensional; the carrier gas is ideal, while its viscosity and thermal conductivity are manifest only in interaction with particles having infinite thermal conductivity; the suspended particles are spheres of a single diameter.

The assumption of one-velocity motion of the disperse particles which is the basis of the C-model of heterogeneous media and the model's lack of a mechanism to prevent two particles from occupying the same point in space lead to several results which are physically incorrect. Among these results are the following: the formation of "sheet"-type surfaces of discontinuity in the particle pseudo-gas [3, 4] and "overlapping" of the cloud of occlusions [5], which are characterized by an unlimited increase in the volume fraction of particles in a certain cross section of the two-phase flow.

Under actual conditions, disperse particles have distributions of velocity and dimensions, which must lead to their collision. The transfer of momentum due to the random motion of the occlusions determines the pressure in the pseudo-gas of solid particles.

Allowing for this pressure in the equations of the aerosol C-model solves the problem of their nonhyperbolicity and the instability of the solutions against small perturbations, and overlapping and sheet discontinuities such as mentioned above become impossible. The random motion of the particles also leads to additional heat release due to interphase friction, while collisions between particles of different sizes results in a redistribution of the kinetic energy associated with their translational motion. However, it is not possible to determine the pressure in a pseudo-gas within the framework of the C-model.

The authors of [6-9] attempted to allow for collisions between particles of different diameters. Thus, in [6], the example of particles of two kinds was used to propose a simple method which reduces to introduction of the effective force acting between two clouds of particles. However, the results obtained in [6] are limited by the assumptions made — specifically, by the assumption that the expected time between two successive collisions between particles is greater than the relaxation time associated with the translational degrees of freedom. This assumption obviously imposes a serious limitation on the mass fraction of the disperse particles.

The authors of [7, 8] described the evolution of polydisperse particles by introducing a distribution function for the velocity of particles of the i-th species. However, the collision integral obtained in these studies in the case of quasi-one-dimensional motion does not satisfy the law of conservation of the total number of particles.

The system of equations obtained in [9] to describe the dynamics of a pseudo-gas of solid particles with allowance for inelastic collisions contains the assumption that the effect of the carrier gas on the motion of the solid particles is negligibly small and the occlusion velocities conform to a Maxwell distribution.

Here, we use the results in [10-12] to propose a continuum-kinetic (CK) model of aerosols, and we study the interaction of a shock wave with an aerosol cloud of finite length.

1. Continuum-Kinetic Model of Aerosols. In the CK-model, the carrier gas is regarded as a continuum, while the evolution of disperse solid particles of the i-th species is described by introducing a velocity distribution function $f_i(t, \mathbf{r}, \mathbf{v}_i)$ whose change is determined by the equation [11, 12]

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$$D_i f_i = \left(\frac{\partial}{\partial t} + \mathbf{v}_i \times \frac{\partial}{\partial \mathbf{r}} + \mathbf{G}_i \times \frac{\partial}{\partial \mathbf{v}_i}\right) f_i = J_{ig} + \sum_{j=1}^{L} J_{ij}. \tag{1.1}$$

Here, L is the number of fractions into which the continuous particle-size distribution is subdivided; J_{ij} is the collision operator corresponding to the interaction between particles of the i-th and j-th species (the Boltzmann form of this operator is valid for elastic and smooth spherical particles); J_{ig} is the collision operator corresponding to interaction of particles of the i-th species and the carrier gas, for which the following diffusion approximation is valid:

$$J_{ig} = \frac{\partial}{\partial \mathbf{v}_i} \times \left\{\frac{\partial}{\partial \mathbf{v}_i} \times (f_i \mathbf{K}_i) - \mathbf{F}_i f_i\right\},$$

$\mathbf{K}_i$ and $\mathbf{F}_i$ are the tensor of the diffusion coefficients in the velocity space and the acceleration of the i-th particle.

Referring all of the quantities in (1.1) to their characteristic values, we obtain the above equation in dimensionless form

$$\varepsilon_i D_i f_i = \beta_i J_{ig} + \sum_{j=1}^{L} J_{ij},$$

where the parameters ε_i and β_i determine the order of the collision operators J_{ij} and J_{ig} and are equal to

$$\varepsilon_i = \frac{\lambda_i}{S}, \quad \beta_i = \frac{\rho}{\rho_i} \frac{U}{U_i} \frac{G_0}{U_i}.$$

Here, λ_i, ρ_i, and U_i are characteristic values of the mean free path, the density of the pseudo-gas, and the velocity of particles of the i-th species; S, G_0, ρ, and U are characteristic values of length, relative velocity between phases, and the density and velocity of the carrier gas.

Relaxation processes in the action of a shock wave on an aerosol cloud are characterized by the fact that the parameters ε_i and β_i change within the ranges: $\varepsilon_+ \lesssim \varepsilon_i \lesssim 1$, $\varepsilon_- \lesssim \beta_i \lesssim 1/\varepsilon_-$ (ε_+ and $\varepsilon_- \ll 1$). This variation is attributable to the appreciable disequilibrium between the phases with respect to translational degrees of freedom during the initial stage of acceleration and to the reduction in the mean free path of the particles as the cloud is compressed.

In connection with this, the Chapman–Enskog method is not applicable over the entire flow region, and it is necessary to use numerical methods to solve Eq. (1.1). Since it is currently not possible to numerically solve the exact kinetic equation, it was proposed in [10] that the BGK-model of this equation be used.

In describing the interaction of shock waves with an aerosol cloud of finite length, we will limit ourselves to the assumptions commonly made for this class of problems. Specifically, we will assume that the motion is unidimensional, the carrier gas is ideal, and its viscosity and thermal conductivity are manifest only in an interaction with particles having infinite thermal conductivity.

Since there is a preferred direction in the problems being examined here, we assume that the velocity components perpendicular to this direction conform to a Maxwell distribution; if we first average the kinetic equation over these components, we obtain the system of equations of the CK-model:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0; \tag{1.2}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x}(p + \rho u^2) = -\sum_{i=1}^{L} \mathscr{F}_{gi}; \tag{1.3}$$

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x}\{u(p + E)\} = \sum_{i=1}^{L} (Q_{if} - q_{gi}); \tag{1.4}$$

$$\frac{\partial E_i}{\partial t} + \frac{\partial}{\partial x}(u_i E_i) = q_{gi}; \tag{1.5}$$

$$\frac{\partial f_i}{\partial t} + \frac{\partial}{\partial x}(v_i f_i) + \frac{\partial}{\partial v_i}(F_{gi} f_i) = -\frac{f_i - f_{i0}}{\tau_i} \tag{1.6}$$

with the condition of normalization of the distribution functions and relations to determine macroscopic parameters of particles of the i-th species

$$N_i = \int f_i dv_i, \ \rho_i = N_i M_i, \ N_i u_i = \int v_i f_i dv_i. \tag{1.7}$$

Here

$$F_{gi} = (\pi d_i^2/8M_i)\,\rho\,(u - v_i)\,|u - v_i|\,C_D; \quad \mathscr{F}_{gi} = N_i M_i < F_{gi} > =$$
$$= M_i \int F_{gi} f_i dv_i; \qquad Q_{if} = -N_i M_i u_i < F_{gi} > + M_i \int c_i f_i F_{gi} dv_i;$$
$$q_{gi} = 2\pi\lambda d_i (T - \Theta_i) N_i \mathrm{Nu}; \ f_{i0} = (N_i/\sqrt{\pi})\alpha^{1/2} \exp\{-\alpha(u_i - v_i)^2\};$$
$$\alpha = 1/(2\langle c_i^2\rangle) = N_i\Big/\Big(2\int f_i c_i dv_i\Big); \quad \tau_i = N_i \sum_{j=1}^{L} \sigma_{ij} \iint f_i f_j\,|v_i - v_j|\,dv_i dv_j;$$

ρ, u, p, E, λ, T are the density, velocity, pressure, total energy per unit volume, thermal conductivity, and temperature of the gas; v_i, c_i, N_i, M_i, d_i, τ_i, σ_{ij}, Θ_i are projections of velocity and the velocity of random motion on the x axis, concentration, mass, diameter, transit time, collision cross section, and surface temperature of particles of the i-th species.

System (1.2)-(1.7) is augmented by the equations of state

$$p = \rho^0 R_g T, \ E = 0.5\rho^0 u^2 + p/(\gamma - 1),$$
$$\rho^0 = \rho\left(1 - \sum_{i=1}^{L} \varphi_i\right), \quad \rho_i = \varphi_i \rho_i^0, \quad E_i = \rho_i \int_{\Theta_0}^{\Theta_i} c_{si} d\Theta,$$

where R_g and γ are the specific gas constant and the adiabatic exponent of the carrier gas; E_i, φ_i, ρ_i^0, c_{si} are the internal energy, volume fraction, density, and heat capacity of particles of the i-th species.

The relations in [13] were used to calculate the drag coefficient C_D and Nusselt number Nu, which enter into the expressions which determine momentum and energy transfer between the particles and the carrier gas.

2. Boundary and Initial Conditions. Numerical Method. It is assumed that at the initial moment of time the shock wave, being a surface of discontinuity, is located at the boundary between the pure gas and aerosol and at subsequent moments of time propagates from left to right. The parameters of the gas phase on each side of the shock wave are connected by the Rankine-Hugoniot relations.

At the initial moment of time, an aerosol cloud of the length S is located to the right of a shock wave in an undisturbed gas under conditions of dynamic and thermal equilibrium. Particle velocity conforms to a Maxwell distribution, with a rms random velocity on the order of 1 m/sec.

To numerically solve the above problem within the framework of the C- and CK-models, we used a conservative monotonic through-counting scheme based on the MacCormack finite difference operator. This is a scheme of the "predictor-corrector" type, and monotonicity is achieved by introducing nonlinear local smoothing without disturbing the conservatism of the difference scheme [14]. The boundary between the pure gas and the aerosol is localized in a manner similar to [15]. The details of the numerical method are discussed in [10].

3. Results of Calculations. To explain characteristic features of the C- and CK-models, we will first examine a simpler case: the interaction of a shock wave with a monodisperse aerosol cloud. The results of calculations show that for small volume and mass fractions ($\varphi < 10^{-4}$, $\varphi_m = \rho_p/\rho < 0.25$), the disperse particles do not have a significant effect on the parameters of the carrier gas, the frequency of collisions, or the velocity associated with the random particle motion. The results of the two models nearly coincide.

An increase in the volume fraction or concentration of particles is accompanied by an increase in the frequency of particle collisions and the mean random velocity. Also, in

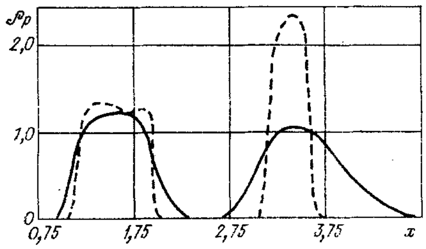

Fig. 1

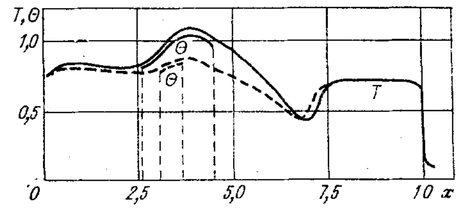

Fig. 2

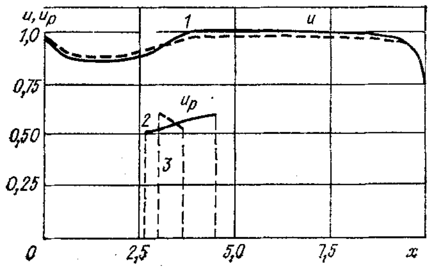

Fig. 3

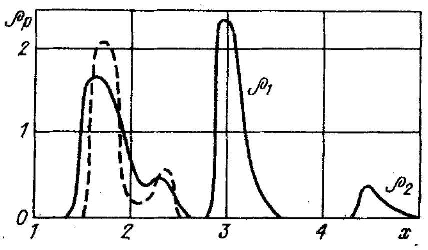

Fig. 4

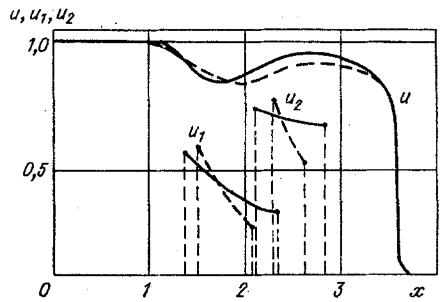

Fig. 5

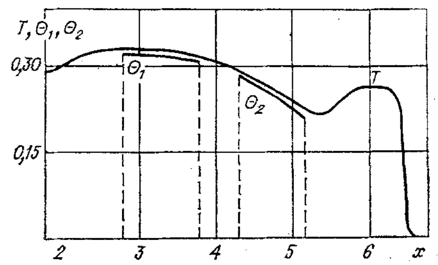

Fig. 6

this case, processes considered only in the CK-model begin to affect the character of relaxation behind the wave front.

Thus, Fig. 1 shows the distribution of the density of an aerosol cloud along the axis at the moments of time $t = 3t_0$, $6t_0$ (t_0 is the time of passage of the shock wave through the cloud) with the initial volume $\varphi = 10^{-3}$ and mass $\varphi_m = 2.5$ fractions of particles of quartz sand with the diameter $6 \cdot 10^{-5}$ m and a Mach number of the incoming shock wave equal to two. Here and below, where necessary, a solid line is used to show values of the variables at the initial moment of time calculated within the framework of the CK-model, while a dashed line is used to show the same in the C-model. All of the variables in the figures are referred to their characteristic values: S — length of the cloud; u_2 — velocity of the wake in the pure gas; S/u_2, $u_2{}^2/R_g$ — characteristic values of time and temperature, respectively. It follows from Fig. 1 that in the CK-model, the stage of initial compression of the particle cloud due to velocity disequilibrium between the phases is replaced by an expansion stage due to the energy of random motion. The C-model shows continuing compression of the cloud.

The nonmonotonic character of the temperature distribution of the carrier gas (Fig. 2) is explained by the presence of two competing processes: interphase friction, leading to heat release; convective heat transfer, causing heating of the particles as a result of heat absorbed from the carrier phase. The maximum of gas temperature is higher in the CK-model because the random motion of the particles leads to additional release of heat as a result of interphase friction. By the moment of time $t = 6t_0$, thermal equilibrium has nearly been achieved between the phases. The temperature of the particle surface turns out to be 20-30% higher in the CK-model than in the C-model.

Figure 3 illustrates the nonmonotonic character of the velocity distribution of the carrier gas (lines 1) at $t = 6t_0$, which is attributable to the retarding effect of the cloud of occlusions and acceleration of the gas due to the heat released from interphase friction.

We should point out the different sign of the gradient of mean particle velocity in the CK- and C-models (lines 2 and 3). This is related to the diffusion of faster-moving particles to the right edge of the cloud as seen in the CK-model.

The effect of polydispersity on the character of relaxation processes behind the wave front can be followed by using the example of a two-fraction particle cloud. Figures 4-6 show results for the case when M = 4.2. The particles, of magnesium oxide, have diameters of $4 \cdot 10^{-5}$ and 10^{-4}.

The distributions of aerosol cloud density shown in Fig. 4 for the moments of time t = 0, $3t_0$, and $6t_0$ illustrate that, over time, a cloud which is initially uniformly mixed separates into two parts under the influence of the force field created by the velocity disequilibrium between the phases. "Overlapping" of the cloud of occlusions is seen in the C-model at $t \geq 3t_0$, and it becomes impossible to perform calculations for subsequent moments of time.

The rapid equalization of the mean velocity of lightweight particles (Fig. 5) in the CK-model is connected with the slowing of occlusions located near the left edge of the cloud as a result of their collisions with slow-moving heavier particles.

Figure 6 shows the dependence of the temperature of the carrier gas and the disperse particles on the longitudinal coordinate. The results for $t = 6t_0$ are shown. The temperature of the fine particles turns out to be lower than the temperature of the coarser particles. This result is valid only for the case when smaller-diameter particles are merely a small addition to the coarser particles.

Analysis of the completed calculations shows that overlapping of the cloud of occlusions does not occur for M = 2-4.5 with a particle diameter $>10^{-4}$ m. In connection with this, we can establish the limit of applicability of the C-model for the given class of problems: small volume and mass fractions ($\varphi < 10^{-4}$, $\varphi_m < 0.25$, when the effect of the disperse phase on the carrier gas is negligible) of occlusions with a diameter greater than 10^{-4} m.

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TWO-PHASE BOUNDARY LAYER WITH AN INCOMPRESSIBLE CARRIER PHASE ON A PLATE, WITH INJECTION AND SUCTION OF GAS FROM THE SURFACE

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Two-phase flows in a boundary layer around bodies of different shapes were examined theoretically in [1-3]. Equations of a two-phase boundary layer were obtained in [1] in four characteristic cases on the basis of asymptotic analysis of the system of equations of two-phase flow at high Reynolds numbers. The structure of a boundary layer with an incompressible carrier phase on the impermeable, stationary surface of a plate was studied in [2]. The investigation [3] examined the effect of the boundary layer on particle trajectory in the flow of an incompressible gas about a sphere in the "single-particle" regime.

Here, we numerically study flow in a two-phase boundary layer about a plate with injection and suction of gas from the surface. An asymptotic analysis of the initial equations of motion of the two-phase medium at high Reynolds numbers produces the boundary condition for the transverse component of particle velocity on the external boundary of the boundary layer.

It was found that the presence of gas suction eliminates the high-particle-density layer in the boundary layer and leads to restructuring of the qualitative flow pattern. An addition is made to the friction coefficient due to particle flow on the surface. With injection of gas from the surface, a layer of pure gas is formed near the surface, while a surface of parameter discontinuity – a sheet – is formed inside the boundary layer.

1. Formulation of the Problem. Written below are the equations of laminar motion of a two-phase mixture in a boundary layer near a flat plate parallel to the incoming flow. We assume that the volume fraction of the chemically inert spherical particles is small, the process is isothermal, there is a small difference between the local characteristics and the mean-volume characteristics, the physical density of the particles is much greater than the density of the carrier phase, Brownian motion of the particles is insignificant, and the Mach numbers are small. The equations of motion in this case have the form [1]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\quad \frac{\partial \rho_s u_s}{\partial x}+\frac{\partial \rho_s v_s}{\partial y}=0, \tag{1.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-\rho_s\frac{c_D}{c_{D0}\sigma}(u-u_s),$$

$$u_s\frac{\partial u_s}{\partial x}+v_s\frac{\partial u_s}{\partial y}=\frac{c_D}{c_{D0}\sigma}(u-u_s),\quad u_s\frac{\partial v_s}{\partial x}+v_s\frac{\partial v_s}{\partial y}=\frac{c_D}{c_{D0}\sigma}(v-v_s).$$

Here, $x = x'/L$, $y = y'/(L\,\mathrm{Re}^{1/2})$ are dimensionless coordinates (the x axis is directed along the plate, while the y axis is directed normal to the plate; $u = u'/u_\infty$, $v = v'/(u_\infty \mathrm{Re}^{1/2})$ are dimensionless components of velocity in the x and y directions, respectively; $\sigma = \tau_s/(L/u_\infty)$ is the Stokes number, characterizing the intensity of viscous interaction of the phases; $\tau_s = \rho_s{}^0 d_s^2/18\mu$ is the characteristic relaxation time of particle velocity; μ is the viscosity coefficient of the carrier phase; $\rho_s{}^0$ is the physical density of the particles;

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